Clarifying Misconceptions about “Investigations in Number, Data, and Space”
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Executive Summary

- There is extensive evidence that using Investigations improves achievement for students when the program is faithfully implemented, especially in the area of problem solving. Investigations lessons were extensively field-tested, revised, rewritten, and retested in actual classrooms for six years with the original edition and for another five years for the second edition. No traditional textbook goes through this type of research and analysis before being published.

- These materials provide a carefully sequenced series of research-based and field-tested investigations which engage the students in guided inquiry. A professional teacher with knowledge of the mathematical goal in mind guides students to understand the concept through skillful questioning and carefully designed instructional settings.

- Because of the emphasis on both oral and written communication in “Investigations in Number, Data, and Space”, students develop the use and understanding of correct mathematical vocabulary much better than in traditionally taught classrooms.

- “Investigations in Number, Data, and Space” advocates treating conventional American algorithms as one of many ways to perform an operation. As with any other strategy, students using an American algorithm should be able to understand and explain why it works. Learning algorithms must not focus on simply memorizing sequences of steps to carry out procedures at the expense of thinking and reasoning to make sense of numerical situations.

- A rigorous mathematical program is defined as one which focuses on the inclusion of all students in challenging and comprehensive instruction. Rigorous instruction includes asking students questions that require higher levels of thinking and probing student answers for clarification and extension. When faithfully implemented, Investigations meets the needs of and challenges all students, imposing no limits on what students may accomplish.

- All children deserve a mathematics education that allows them to build concepts and skills on a firm foundation of prior knowledge and connections to their world; supports and challenges them to develop skill, persistence, confidence and independence in problem solving; enables them to articulate, defend and question mathematical ideas; and engages them in a community of learners. This is what constructivist learning entails.

- The goal of NCTM’s Curriculum Focal Points is to encourage curriculum developers to reject lengthy lists of isolated skills and, instead, to provide students with a connected, coherent, and expanding body of mathematical knowledge and mathematical ways of thinking – exactly what the authors of “Investigations in Number, Data, and Space” have done.

- Any comprehensive mathematics program in the 21st century should help students learn to use calculators, computers, and other technological tools as a part of learning mathematics, as Investigations does.

- The crisis in mathematics education in the United States is at least twenty-five years old. Programs like Investigations did not create the problem, but were developed after 1989 to address the problem.
Math instruction cannot be effective if it is based on extreme positions. ... U.S. students need more skill and more understanding along with the ability to apply concepts to solve problems, to reason logically, and to see math as sensible, useful, and doable. Anything less leads to knowledge that is fragile, disconnected, and weak. (“Helping Children Learn Mathematics,” p. 12)

Critics’ Assertion: The track record for Investigations is not impressive.

The TERC web site continues to maintain a link for validation and research on its home page which includes the five studies he referenced. (http://investigations.terc.edu/)

One of these studies, the ARC Tri-state Achievement Study, was a large scale study which looked at the performance of students using standards-based curricula (i.e. “Investigations in Number, Data, and Space”, “Everyday Math,” and “Math Trailblazers”) on state mandated standardized tests in Massachusetts, Illinois, and the state of Washington and compared these results to matched schools that were using a traditional approach. The study included over 100,000 students, 51,340 students who had studied math using standards-based textbook materials for at least two years and 49,535 students from non-using comparison schools matched by reading level, socioeconomic status, and other variables. This matching routine was carried out for each of the state-grade combinations in order to identify a set of comparison schools that had not implemented any one of the three standards-based programs, but that were similar in how they would be expected to perform on the respective statewide tests. The matching procedure selected one matched comparison school using traditional materials for each of the 742 eligible schools used in the analysis. Results show that the average mathematics scores of students in schools using standards-based materials are significantly higher than the average scores of students in their matched comparison schools. The results hold across five different state-mandated tests, and across topics ranging from computation, measurement, geometry, and algebra to problem solving and making connections. The results also hold across all income and racial/ethnic subgroups, except for Hispanic students, where there are no significant differences between the scores. There are many examples of school districts that implemented “Investigations in Number, Data, and Space” with success over time. One notable example is Boston City Schools. A report of the 2005 NAEP (National Assessment of Educational Progress) scores showed that Boston improved their scores in mathematics more rapidly than the other eleven urban school systems whose scores are publicly reported. Boston also clearly had the greatest score increase for 8th graders from 2003-2005, a greater gain than any of the other urban areas, and across the nation as a whole. These eighth grade students would have used “Investigations in Number, Data, and Space” in elementary school. This school district has a long history of using “Investigations in Number, Data, and Space”. Further information about NAEP results can be found by visiting the link below:

Many districts using “Investigations in Number, Data, and Space” have demonstrated success as measured by their own state’s testing program.

Together, the studies of “Investigations in Number, Data, and Space” indicate that:
• Students using “Investigations in Number, Data, and Space” do as well or better than students using other curricula in straight calculation problems involving basic facts and the whole number operations.
• Students using “Investigations in Number, Data, and Space” have a better understanding of number and number relationships than students working with more traditional programs.
• “Investigations in Number, Data, and Space” works equally well with students at different achievement levels in mathematics.
• Students who use “Investigations in Number, Data, and Space” achieve greater accuracy on word problems and on more complex calculations than students in comparison classrooms.

Critics’ Assertion: **Investigations is discovery learning.**

One of the criticisms about “Investigations in Number, Data, and Space” is that it is “discovery” math where students “discover” concepts on their own. Discovery learning has various definitions. At one end of the spectrum we find discovery learning in its simplest form. The tools and information needed to solve a problem or learn a concept are provided and the learner “makes sense” of them. Another definition is discovery learning as experimentation with some extrinsic intervention—clues, coaching, and a framework to help learners get to a reasonable conclusion. At the other end of the continuum is the expository teaching model of discovery learning where the learner “discovers” what the teacher decides he is to discover using a process prescribed by the teacher. (*Encyclopedia of Educational Technology*)

In “Investigations in Number, Data, and Space,” “…the focus is on mathematical reasoning and problem solving in a true sense. The program provides a carefully sequenced series of research-based and field-tested investigations, which engage the students in guided inquiry. It emphasizes that in order to solve problems, students must learn to describe, to compare, and to discuss their approaches. ... Multiple strategies are encouraged, and communication about mathematics is central. Students explore the central topics in depth through a series of investigations, gradually encountering and using many important mathematical ideas.” (Economopolous, et al) An informed teacher guides student learning so that the student moves to a more sophisticated and complex understanding of the mathematical concept and becomes more proficient in using this concept to solve more complex problems. Hence guided inquiry is not “discovery” learning in the extreme definition of the term. Rather, a professional teacher with knowledge of the mathematical goal in mind guides students to understand the concept through skillful questioning and carefully chosen instructional settings.

Critics’ Assertion: **Investigations does not teach standard terminology, formulas, or American algorithms.**

Standard terminology:
“Investigations in Number, Data, and Space” does not reject the use of standard terminology. Rather, it deals with terminology in the context of the mathematics. In the student handbook in the revised version, the “math words” critical to each concept are stated in a box at the top of the page. Vocabulary should not be taught without context, and the Investigations series follows this principle. Because of the emphasis on both oral and written communication in “Investigations in Number, Data, and Space”, students develop the use and understanding of correct mathematical vocabulary much better than in traditionally taught classrooms.
Algorithms:
“Investigations in Number, Data, and Space” advocates treating the conventional American algorithm for each operation as just one of many ways to perform the operation. As with any other strategy, students using an American algorithm should be able to understand and explain why it works. Learning algorithms must not focus on simply memorizing sequences of steps to carry out procedures at the expense of thinking and reasoning to make sense of numerical situations. Students should select from a variety of strategies to find the most efficient way to solve a particular problem and keep track of multiple steps in a problem. For example, to compute mentally, 26 + 26 may be thought of as 25 + 25 + 2.

Critics’ Assertion: *Investigations* lacks rigor.
Rigor is a term often used by educators without much time or attention paid to its definition as it applies to education. Thus, the question, what is meant by rigor? In relation to mathematics it is the expectation of systematic reasoning or proof. In other words, it is expected that the student will have the ability to make an argument with sound mathematical reasoning. The crux behind this idea is that it helps the student and teacher identify, and thus learn to avoid, fallible arguments. In the No Child Left Behind Act, a rigorous mathematical program is defined as one which focuses on the inclusion of all students in challenging and comprehensive instruction using best practices and techniques. In an alternate definition, mathematical rigor is achieved when students can comprehend the distinctions and relationships between all the systems they are studying. “Investigations in Number, Data, and Space” meets each of these definitions. To demonstrate this, the following is an alignment of a few of the “Rigorous Instruction and Thinking Skills” as outlined in “Leading the Learning” (Rutherford, 2002) and the methods/practices of “Investigations in Number, Data, and Space.”

- **Rigorous instruction must ask all students questions that require higher levels of thinking and probe student answers for clarification and extension.**
  In each of the activities in “Investigations in Number, Data, and Space” it is not enough that the student arrive at a correct answer. The student must be able to clarify their thinking. They must be able to articulate the mathematical reasoning behind their solution, and be able to justify its accurateness. This allows teachers to ask questions to not only further the thinking of that particular student, but of all students.

- **Instruction must name, model, and provide practice of thinking processes so students can build and independently access their own thinking skills repertoire.** “Investigations in Number, Data, and Space” is built upon students developing their own thinking skills and repertoire. They are encouraged to solve problems in ways which are mathematically sound and which have meaning to them. In doing this, the rote memorization of 7 x 8 = 56 is not just a fact they know, but one they can access quickly AND has meaning. For example, they know that it is seven sets of eight that is 56 units, or perhaps they see it visually as seven rows of eight squares (area). They come to understand that seven eights yields the same result as eight sevens, and eventually generalize the commutative property for multiplication. The students can also argue that it is 7 x 4 + 7 x 4 = 28 + 28 = 56. Mathematicians recognize that, in this process, the students are developing the distributive property concept [7 x 4 + 7 x 4 = 7(4 + 4)] on their own. Thus, students will be able to access the meaning of properties later based on their own development and investigation of the concept.

- **Students should be taught the skills of inquiry, dialogue and debate.**
  In “Investigations in Number, Data, and Space”, students are continually expected to
discuss/dialogue about their thinking. For example, in an activity called “Number of the Day”, students talk about the various ways they arrived at the number. With the guidance of the teacher, they begin to identify how ideas are alike and different, as well as how they relate to one another. This allows for inquiry and debate of the reasoning behind the responses. Over time, students begin to recognize that there are many effective methods. More importantly, they learn which methods are most efficient for them based on their own strengths and reasoning skills, and which methods work best depending on the combinations of the numbers involved. The program allows students to go outside of the widely known boxed algorithm to find the strategyestrategies that work best, are most efficient, and have meaning for them.

It is significant that the Advanced Placement, International Baccalaureate, and Cambridge courses and programs require students to do exactly what “Investigations in Number, Data, and Space” is asking of them. In upper level math problems, students must first find a method/strategy for solving the problem. It must be one which they understand and can present logically, using and supporting with each step sound mathematical reasoning. They must justify their reasoning, and usually have an alternate method, or argument, to demonstrate the accuracy of their answer. This type of reasoning goes far beyond memorizing a fact, or walking blindly through the steps of a memorized procedure. It requires true synthesis of the material by each and every student; all students must have sound mathematical understanding and reasoning, both of which they must have developed with guidance through contexts which make sense to them. Students are asked to demonstrate the levels of synthesis and evaluation in these courses through free response testing and culminating portfolios. In these assessments (for which the methods of “Investigations in Number, Data, and Space” initiates preparedness), the ability to demonstrate sound mathematical reasoning, to demonstrate thinking at the level of evaluation in Bloom’s Taxonomy, and to do so in a way that is most efficient for the individual is a challenge. Therein lies the rigor for ALL students.

Critics’ Assertion: Constructivist learning is an inferior teaching method.

Constructivism is a theory about how students learn that is not new. It has been supported historically by the work of Dewey, Montessori, Piaget, Bruner, and Vygotsky, and is further supported by recent research in cognitive psychology. Over 30 years of results from state, national, and international assessments indicate that U.S. students can generally perform straightforward computational procedures, but they tend to have a more limited understanding of fundamental mathematical ideas and many have difficulty applying mathematical skills to solve problems in various contexts (see NAEP and TIMSS reports). Skills-focused traditional approaches are not helping us achieve the goal of mathematical proficiency for all students.

Publications of the National Research Council provide support for the type of mathematics instruction supported by “Investigations in Number, Data, and Space”. (The National Research Council, the principal operating agency of the National Academy of Sciences and the National Academy of Engineering, is a private, nonprofit institution that provides science advice under a congressional charter. All of its publications are available for free on the Web.)

“Adding It Up: Helping Children Learn Mathematics” (National Research Council, 2001) and the short book, “Helping Children Learn Mathematics” (NRC, 2002), included with this paper, describe five “intertwined and equally-important” strands that comprise mathematical proficiency: reasoning, applying, understanding, computing, and engaging. It makes the case that instruction which develops proficiency for all students provides carefully planned and sequenced instruction, creates communities of learners and encourages student independence and confidence.
“How Students Learn: History, Mathematics, and Science in the Classroom” by M. Suzanne Donovan, John D. Bransford (National Research Council, 2005) articulates that effective learning environments - whether in history, mathematics, or science - must be simultaneously learner-centered, knowledge-centered, assessment-centered, and community-centered:

- The learner-centered lens encourages attention to preconceptions, and begins instruction with what students think and know.
- The knowledge-centered lens focuses on what is to be taught, why it is taught, and what mastery looks like.
- The assessment-centered lens emphasizes the need to provide frequent opportunities to make students’ thinking and learning visible as a guide for both the teacher and the student in learning and instruction.
- The community-centered lens encourages a culture of questioning, respect, and risk taking.

“Adding It Up: Helping Children Learn Mathematics” and “How Students Learn: History, Mathematics, and Science in the Classroom” have their roots in the report of the Committee on Developments in the Science of Learning, “How People Learn: Brain, Mind, Experience and School” (National Research Council 1999), recognized as an authoritative synthesis of decades of research in the cognitive and developmental learning sciences.

In December 2006, the New Commission on the Skills of the American Workforce released its report, “Tough Choices or Tough Times,” recommending the total revamping of the American education system. Among its recommendations: “Develop standards, assessments, and curriculum that reflect today’s needs and tomorrow’s requirements.” According to the report, schools need to develop and test not only discipline-based knowledge, but other vital qualities such as “creativity and innovation, facility with the use of ideas and abstractions, the self-discipline and organization needed to manage one’s own work and drive it through to a successful conclusion, the ability to function well as a member of a team, and so on.” (p. 14, Executive Summary: http://skillscommission.org/executive.htm)

All children deserve a mathematics education that allows them to build concepts and skills on a firm foundation of prior knowledge and connections to their world; supports and challenges them to develop skill, persistence, confidence and independence in problem solving; enables them to articulate, defend and question mathematical ideas; and engages them in a community of learners. These respected and research-based reports, produced by organizations outside the field of education, clearly support the goals of “Investigations in Number, Data, and Space” as vital to the futures of our children and our nation.


The release in September, 2006, of the National Council of Teachers of Mathematics’ “Curriculum Focal Points for Pre-kindergarten through Grade 8 Mathematics” was erroneously hailed by critics of conceptually-based mathematics instruction as a “remarkable reversal” of NCTM’s position as
presented in the previous Standards documents. Instead, the publication is intended to bring more coherence to the very diverse mathematics curricula currently in use. It provides a framework for states and districts to design more focused curricular expectations and assessments for pre-K to grade 8 mathematics curriculum development and for textbook publishers to avoid the “mile-wide, inch-deep” mathematics coverage that often results from attempts to satisfy varied state requirements.

http://www.nctm.org/focalpoints/

“Curriculum Focal Points for Pre-kindergarten through Grade 8 Mathematics” provides descriptions of the most significant mathematical concepts and skills at each grade level, organizing the rest of the curriculum around these described focal points, with a clear emphasis on the processes that NCTM’s “Principles and Standards” addresses: communication, reasoning, representation, connections, and, particularly, problem solving. The goal is to encourage curriculum developers to reject lengthy lists of isolated skills and, instead, to provide students with a connected, coherent, and expanding body of mathematical knowledge and mathematical ways of thinking – exactly what the authors of “Investigations in Number, Data, and Space” have done. In fact, in the 2nd edition of “Investigations in Number, Data, and Space,” connections to the “Curriculum Focal Points” are shown in the materials.

An example focal point from “Curriculum Focal Points and Connections for Grade 4” illustrates that the “Curriculum Focal Points” do not represent a rejection of the Investigations philosophy:

**Number and Operations and Algebra: Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication:**

Students use understandings of multiplication to develop quick recall of the basic multiplication facts and related division facts. They apply their understanding of models for multiplication (i.e., equal sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multidigit whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for multiplying whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.

Contrary to the accusations of its critics, computational fluency has been a goal of “Investigations in Number, Data, and Space” since its inception. Basic facts are an essential component of computational fluency and are a focus of activities and games throughout the program. The authors do take a stance against the arbitrary teaching of algorithms in the absence of conceptual understanding. Students utilize their developing number sense and place value understandings to apply computational processes in the context of problems. The role of the teacher is to nudge them toward ever-greater efficiency. No algorithm is rejected. Indeed, the 2nd Edition of “Investigations in Number, Data, and Space” explicitly explores the U.S. traditional (digit-based) algorithms after the students have developed a firm number-sense understanding of processes. The approach to computational fluency in “Investigations in Number, Data, and Space” is fully consistent with that articulated in the “Curriculum Focal Points.”

**Critics’ Assertion: Investigations uses a heavy reliance on calculators.**
In “Investigations in Number, Data, and Space” calculators are used as a means to develop conceptual understanding and not as a substitute for computational fluency. For example, students may record the position of the decimal point after various examples involving decimal computation using a calculator and then propose generalizations from their observations. Any comprehensive mathematics program in the 21st century should help students learn to use calculators, computers, and other technological tools as a part of learning mathematics. The “Principles and Standards for School Mathematics” documents make it clear, however, that such tools do not replace the need to learn basic facts, to compute mentally, or to do reasonable paper-and-pencil computation.

**Critics’ Assertion: California rejected reform math after a twelve year experiment in 1997.**

To fully respond to this statement requires some background information regarding both NCTM’s Standards of 1989 (revised in 2000) and the politics of California at the time. The reality is that the crisis in mathematics education is at least twenty-five years old. This crisis became part of the national agenda with such publications as “An Agenda For Action” (NCTM, 1980), “A Nation at Risk” (National Commission of Excellence in Education, 1983) and “Everybody Counts: A Report to the Nation on the Future of Mathematics Education” (NRC, 1989). It was at the end of that decade that the National Council of Teachers of Mathematics released their “Curriculum and Evaluation Standards for School Mathematics” (1989). Shortly after their publication, the National Science Foundation began funding development of large scale, multi-grade instructional materials in mathematics to support the realization of the NCTM Standards in the classroom. “Investigations in Number, Data, and Space” was one of these programs. “Programs like “Investigations in Number, Data, and Space’ were developed to address the crisis in mathematics education. They did not cause the problem, as the anti-reform people would have us believe. They were the solution to the problem; a problem that has existed for many years.” (Sherry Fraser, National Council of Supervisors of Mathematics, recent testimony at the National Math Advisory Panel)

California revised its state standards in 1992 in an effort to align itself with NCTM’s standards of 1989. The claim that the 1992 framework had failed its elementary students was widespread by early 1995. The reality is very little "reform" mathematics was actually taking place across the state, since districts had the option of using traditional texts and many did continue to do just that. However, a movement toward standards-based instruction was happening, but was in essence derailed in 1997. In that year, California developed a new set of standards which focused on basic computation skills and a procedural approach to teaching mathematics. This change occurred by and large because of the significant influence of four Stanford University mathematics professors who essentially rewrote the proposed standards to make them “Mathematically Correct.” One might note that this happens to be the title of a public Web site whose primary mission is to keep mathematics education as it has always been. The voices of K-12 educators and educational researchers were not heard in the revision of these standards in California.

There is now data to show these California standards of 1997 haven’t improved math education at all. Most of California’s students today have had all of their instruction based on these standards since they were adopted almost ten years ago. A visit to the California Department of Education’s website on testing (http://star.cde.ca.gov/star2006/viewreport.asp) will reveal that their results have been anything but exemplary. Their 2006 data shows that only 23% of students are proficient in Algebra I by the end of high school, a gain of two points over four years. At the Algebra II level, only 45% of California’s students actually take the course and only 25% of those are proficient. Three years of college preparatory mathematics is required for entrance into California’s colleges and universities, yet less than 12% of California’s high school graduates now have the minimum proficiencies expected by
higher institutions. And these numbers don’t even take into account the 30% of students who drop out of high school. “California is one state we do not want to emulate or look to for solutions to the problems in mathematics education.” (Sherry Frazer, National Council of Supervisors of Mathematics, recent testimony to the National Math Advisory Panel, November, 2006)

We highly recommend reading the following online article available at the Kappan web site which will give further insight into the politics of California and the effect it had on mathematics education in that state. (https://www.pdkintl.org/kappan/kbec0003.htm)

We also recommend reading a recent report put out by Education Trust, Inc. It is called “Education Watch: Key Education Facts and Figures, Achievement, Attainment, and Opportunity, From Elementary School through College.” (http://www2.edtrust.org/edtrust/summaries2006/states.html)

This report clearly shows that Virginia is doing far better than California in terms of closing the achievement gap in almost every area. It should also be noted that Massachusetts, where standards-based curricula are widely used is outperforming both Virginia and California overall on the NAEP test. (http://nces.ed.gov/nationsreportcard/pdf/dst2005/2006457r.pdf)

Indeed, a close look at our own middle school and high school data at the local and state levels indicates there is much progress yet to be made in the area of mathematics education, and this progress calls for change in the status quo.

We believe that our students deserve a better mathematics future. There is truth to the adage that if one continues to do the same thing, then the results will remain unchanged. We want our students to do the “basics,” but we want much more. This is clearly stated in the Kings Local School District mathematics curriculum. We want students to be able to reason and think deeply about the mathematical content in which they are engaged, as well as communicate and represent their thinking both orally and in writing. This is what “Investigations in Number, Data, and Space” does for students. The mathematics content they will learn remains unchanged, but the way in which they learn that content is different than the ways some teachers and parents learned mathematics. “Investigations in Number, Data, and Space” has been through rigorous development over many years that included design, piloting, redesign, field-testing, redesign, and publication. This amount of careful development and evaluation is rarely seen in regular textbook production.

It is important to remember that we are not merely preparing students to pass the Ohio Achievement Test at their particular grade level. We are preparing them for higher-level mathematics that they will study in later grades. We are preparing them to be mathematically literate citizens. “Investigations in Number, Data, and Space” is a crucial part of King’s commitment to enable all children to understand and use mathematics to solve problems for life.